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PJ-276

100217

I Semester M.Sc. Examination, February - 2020
(CBCS - Y2K17/Y2K14 Scheme)

MATHEMATICS

M105T : Discrete Mathematics

Time : 3 Hours

Max. Marks : 70

Instructions : *(i) Answer* ***any five*** *full questions.*
(ii) ***All*** *answers carry equal marks.*

1. (a) Test the validity of the following argument : **6+4+4**

If I study, I will not fail in the examination.

If I do not watch TV in the evenings, I will study.

I failed in the examination.

∴ I must have watched TV in the evenings.

(b) Give an indirect proof of the statement :

"Let n be an integer. If n^2 is odd then n is odd".

(c) By the method of contradiction disprove the statement :

"The sum of two odd integers is an odd integer".

2. (a) Explain the product rule and the sum rule of counting techniques. Also explain the inclusion-exclusion principle. **6+4+4**

(b) A computer company receives 350 applications from computer graduates for a job planning a line of new web browsers. Suppose that 220 of these people majored in computer science, 147 are majored in business and 51 majored in both computer science and business. Using the inclusion-exclusion principle determine how many of these applicants majored neither in computer science nor in business.

(c) Find the coefficient of xyz^2 in the expansion of $(2x-y-z)^4$.

3. (a) Explain how Fibonacci numbers are modelled by a recurrence relation.

(b) Solve $U_{n+2}-4U_{n+1}+4U_n=(n+1)^2$; $U_0=0$, $U_1=1$. **5+4+5**

(c) Using generating function solve the following difference equation :

$F_n=F_{n-1}+F_{n-2}(n\geq 2)$; $F_0=1$, $F_1=1$.

P.T.O.

4. (a) Discuss the nature of the relation from A to B where $A=\{1, 2, 3\}$, $B=\{1, 2, 3, 4\}$ and $R=(x, y)/5x+2y$ is a prime number}. Write down the relation matrix and digraph corresponding to R. Write down the number of in-vertices and out-vertices. Comment on the relation R. **5+5+4**

(b) Determine the reflexive, symmetric and transitive closures of a relation R whose matrix is given by :

$$M_R=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 1\\0 & 1 & 1\end{bmatrix}$$

(c) Write the Hasse diagram for the set D_{20} which consists of all divisors of 20 and $R=\{(x, y) : x, y \in D_{20}$ and x divides $y\}$.

5. (a) Define a bipartite graph. Prove that a graph is bipartite if and only if it contains no odd cycle. **5+4+5**

(b) Define graph isomorphism. Verify that the following graphs are isomorphic or not.

(c) Define self complementary graphs. Prove that any self complementary graph has 4n or 4n+1 vertices for $n \geq 1$.

6. (a) Define an Eulerian graph. Prove that a connected graph G is an Eulerian graph if and only if G can be decomposed into edge-disjoint-cycles. **4+7+3**

(b) Define a Hamiltonian graph. State and Prove Ore's theorem for Hamiltonian graphs.

(c) Write a short note on travelling salesman problem.

7. (a) Define a planar graph. Show that K_5 and $K_{3, 3}$ are non-planar graphs. Further, state and prove Euler's polyhedron formula. **6+6+2**

(b) With usual notation, show that $\alpha_0(G)+\beta_0(G)=P=\alpha_1(G)+\beta_1(G)$.

(c) Define vertex and edge connectivity of a graph with an example.

8. (a) Show that a graph G is a tree if and only if every two vertices of G are connected by a unique path. **5+5+4**

(b) Define binary tree with an example. Prove the following binary tree with $p \geq 3$ vertices.

(i) The number of vertices is always odd.

(ii) The number of pendent vertices is $\frac{p+1}{2}$.

(c) Define minimal spanning tree. Explain Krushkal's algorithm with an example.

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